Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Heat and Mass Transfer 48 (2005) 4460–4466

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity

S. Mukhopadhyay, G.C. Layek *, Sk.A. Samad

Department of Mathematics, The University of Burdwan, Burdwan 713104, W.B., India

Received 28 February 2005; received in revised form 21 April 2005

Abstract

This paper concerns with a steady two-dimensional flow of an electrically conducting incompressible fluid over a heated stretching sheet. The flow is permeated by a uniform transverse magnetic field. The fluid viscosity is assumed to vary as a linear function of temperature. A scaling group of transformations is applied to the governing equations. The system remains invariant due to some relations among the parameters of the transformations. After finding two absolute invariants a third-order ordinary differential equation corresponding to the momentum equation and a second-order ordinary differential equation corresponding to energy equation are derived. The equations along with the boundary conditions are solved numerically. It is found that the decrease in the fluid viscosity makes the velocity to decrease with the increasing distance of the stretching sheet. At a particular point of the sheet the fluid velocity decreases with the decreasing viscosity but the temperature increases in this case. It is found that with the increase of magnetic field intensity the fluid velocity decreases but the temperature increases at a particular point of the heated stretching surface. The results thus obtained are presented graphically and discussed.

Keywords: Scaling groups; Stretching sheet; Temperature-dependent fluid viscosity; Magnetic field

1. Introduction

Lie-group analysis, also called symmetry analysis was developed by Sophius Lie to find point transformations which map a given differential equation to itself. This method unifies almost all known exact integration techniques for both ordinary and partial differential equations [7]. Group analysis is the only rigorous mathematical method to find all symmetries of a given differential equation and no ad hoc assumptions or a prior knowledge of the equation under investigation is needed. The boundary layer equations are especially interesting from a physical point of view because they have the capacity to admit a large number of invariant solutions i.e. basically analytic solutions. In the present context, invariant solutions are meant to be a reduction to a simpler equation such as an ordinary differential equation (ODE). Prandtl's boundary layer equations admit more and different symmetry groups. Symmetry groups or simply symmetries are invariant transformations which do not alter the structural form of the equation under investigation [4].

The non-linear character of the partial differential equations governing the motion of a fluid produces

* Corresponding author. Tel.: +91 342 2557741; fax: +91 342 2530452.

E-mail address: glayek@yahoo.com (G.C. Layek).

0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2005.05.027

Nomenclature

A	fluid viscosity variation parameter
B_0	strength of uniform magnetic field
F	non-dimensional stream function
F^*	variable
F'	first-order derivative with respect to η
F''	second-order derivative with respect to η
F'''	third-order derivative with respect to η
G	absolute invariant defined in $G = x^r\psi^*$
M	non-dimensional magnetic parameter
Pr	Prandtl number
p, q	variables
T	temperature of the fluid
T_w	temperature of the wall of the surface
T_∞	free-stream temperature
u, v	components of velocity in the x- and y-directions
z	variable

Greek symbols

$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha', \alpha''$	transformation parameters
β', β''	transformation parameters
η	similarity variable
Γ	Lie-group transformations
κ	the coefficient of thermal diffusivity
μ	dynamic viscosity
μ^*	reference viscosity
ν^*	reference kinematic viscosity
ψ	stream function
ψ^*	variable
ρ	density of the fluid
σ	conductivity of the fluid
θ	non-dimensional temperature
θ^*, $\bar{\theta}$	variables
θ'	first-order derivative with respect to η
θ''	second-order derivative with respect to η

difficulties in solving the equations. In the field of fluid mechanics, most of the researchers try to obtain the similarity solutions in such cases. In case of scaling group of transformations, the group-invariant solutions are nothing but the well known similarity solutions [6]. A special form of Lie-group of transformations, known as scaling group, is used in this paper to find out the full set of symmetries of the problem and then to study which of them are appropriate to provide group-invariant or more specifically similarity solutions.

The study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. There has been a great interest in the study of magnetohydrodynamic flow and heat transfer in any medium due to the effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow has attracted the interest of many researchers due to its applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions. By the application of magnetic field, hydromagnetic techniques are used for the purification of molten metals from non-metallic inclusions. So such type of problem, that we are dealing with, is very much useful to polymer technology and metallurgy. Crane [5] extended the work of Sakiadis [1,2] who was the first person to study the laminar boundary layer flow caused by a rigid surface moving in its own plane. Gupta and Gupta [9] studied the problem in the light of suction or blowing. In all the above mentioned studies, fluid viscosity was assumed uniform in the flow region. But it is known from physics that with the rise of temperature, the coefficient of viscosity decreases in case of liquids whereas it increases in case of gases. Abel et al. [8] studied the visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity.

In this paper, application of scaling group of transformation for a hydromagnetic flow over a heated stretching sheet with variable viscosity has been employed. This reduces the system of non-linear coupled partial differential equations governing the motion of fluid into a system of coupled ordinary differential equations by reducing the number of independent variables. The system remains invariant due to some relations among the parameters of the transformations. Two absolute invariants are obtained and used to derive a third-order ordinary differential equation corresponding to momentum equation and a second-order ordinary differential equation corresponding to energy equation. Using shooting method the equations are solved. Finally, analysis have been made to investigate the effect of fluid viscosity parameter, Prandtl number and magnetic parameter in the motion of an electrically conducting liquid.

2. Equations of motion

We consider the steady two-dimensional flow of a viscous incompressible electrically conducting fluid over a heated stretching sheet in the region $y > 0$. Keeping the origin fixed, two equal and opposite forces are applied along the x-axis which results in stretching of the sheet and a uniform magnetic field of strength B_0 is imposed along the y-axis.

The continuity, momentum and energy equations governing such type of flow are written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\kappa\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the components of velocity, respectively, in the x- and y-directions, T is the temperature, κ is the coefficient of thermal diffusivity, ρ is the fluid density (assumed constant), σ is the conductivity of the fluid, μ is the coefficient of fluid viscosity.

2.1. Boundary conditions

The boundary conditions are given by

$$u=cx,\quad v=0,\quad T=T_w \quad \text{at } y=0. \tag{4}$$

$$u\to 0,\quad T\to T_\infty \quad \text{as } y\to\infty. \tag{5}$$

Here $c(>0)$ is a constant, T_w is the uniform wall temperature, T_∞ is the temperature far from the sheet.

2.2. Method of solution

We now introduce the following relations for u, v and θ as

$$u=\frac{\partial \psi}{\partial y},\quad v=-\frac{\partial \psi}{\partial x}, \tag{6}$$

and

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \tag{7}$$

where ψ is the stream function.

The temperature dependent viscosity is given by Batchelor [3]

$$\mu=\mu^*[a+b(T_w-T)], \tag{8}$$

where μ^* is the constant value of the coefficient of viscosity far away from the sheet and a, b are constants and $b>0$.

We have used viscosity-temperature relation $\mu=a-bT(b>0)$ which agrees quite well with the relation $\mu=1/(b_1+b_2T)$ [10,12] and also with the relation $\mu=e^{-aT}$ [11] when second and higher order terms neglected in the expansions.

The range of temperature i.e. (T_w-T_∞) studied here is 0–23 °C.

The coefficient of viscosity μ of a large number of liquids agree very closely with the emperical formula given by $\mu=c/(a+bT)^n$ where a, b, c, n are constants depending on the nature of liquid. This agrees well with $n=1$ for pure water with our formulation for fluid viscosity.

Taking the relations (6) and (7) into consideration in the boundary layer equation (2) and the energy equation (3), we get the following equations

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=-Av^*\frac{\partial\theta}{\partial y}\frac{\partial^2\psi}{\partial y^2}+v^*(a+A(1-\theta))\frac{\partial^3\psi}{\partial y^3}-cM^2\frac{\partial\psi}{\partial y} \tag{9}$$

and

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\kappa\frac{\partial^2\theta}{\partial y^2}, \tag{10}$$

where $A=b(T_w-T_\infty)$, $v^*=\frac{\mu^*}{\rho}$ and $\frac{\sigma B_0^2}{\rho}=cM^2$, M is the Hartman number.

The boundary conditions (4) and (5) then become

$$\frac{\partial\psi}{\partial y}=cx,\quad \frac{\partial\psi}{\partial x}=0,\quad \theta=1 \quad \text{at } y=0, \tag{11}$$

$$\frac{\partial\psi}{\partial y}\to 0,\quad \theta\to 0 \quad \text{as } y\to\infty. \tag{12}$$

2.3. Application of scaling group of transformations

We now introduce the simplified form of Lie-group transformations [13]

$$\Gamma: x^*=xe^{\epsilon\alpha_1},\quad y^*=ye^{\epsilon\alpha_2},\quad \psi^*=\psi e^{\epsilon\alpha_3},$$
$$u^*=ue^{\epsilon\alpha_4},\quad v^*=ve^{\epsilon\alpha_5},\quad \theta^*=\theta e^{\epsilon\alpha_6}. \tag{13}$$

Eq. (13) may be considered as a point-transformation which transforms co-ordinates (x,y,ψ,u,v,θ) to the co-ordinates $(x^*,y^*,\psi^*,u^*,v^*,\theta^*)$.

Substituting (13) in (9) and (10) we get,

$$e^{\epsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial y^{*2}}\right)$$
$$=-Av^*e^{\epsilon(3\alpha_2-\alpha_3-\alpha_6)}\frac{\partial\theta^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial y^{*2}}+(a+A)v^*e^{\epsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\psi^*}{\partial y^{*3}}$$
$$-v^*A\theta^*e^{\epsilon(3\alpha_2-\alpha_3-\alpha_6)}\frac{\partial^3\psi^*}{\partial y^{*3}}-cM^2e^{\epsilon(\alpha_2-\alpha_3)}\frac{\partial\psi^*}{\partial y^*}, \tag{14}$$

$$e^{\epsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x}\frac{\partial\theta^*}{\partial y^*}\right)$$
$$=\kappa e^{\epsilon(2\alpha_2-\alpha_6)}\frac{\partial^2\theta^*}{\partial y^{*2}}. \tag{15}$$

The system will remain invariant under the group of transformations Γ, so we would have the following relations among the transformation parameters, namely

$$\alpha_1+2\alpha_2-2\alpha_3=3\alpha_2-\alpha_3-\alpha_6=3\alpha_2-\alpha_3=\alpha_2-\alpha_3 \tag{16}$$

and

$$\alpha_1+\alpha_2-\alpha_3-\alpha_6=2\alpha_2-\alpha_6. \tag{17}$$

From the relation $3\alpha_2-\alpha_3=\alpha_2-\alpha_3$, we get $\alpha_2=0$.

The relation $3\alpha_2-\alpha_3-\alpha_6=3\alpha_2-\alpha_3$, we get $\alpha_6=0$.

Again from the relation $\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3$, we get $\alpha_1 = \alpha_3$(since $\alpha_2 = 0$). The relations $u^* = \frac{\partial\psi^*}{\partial y^*}$ and $v^* = -\frac{\partial\psi^*}{\partial x^*}$ gives us $\alpha_3 = \alpha_4$, $\alpha_5 = 0$. Thus we get $\alpha_1 = \alpha_3 = \alpha_4$; $\alpha_2 = \alpha_5 = \alpha_6 = 0$.

From the boundary conditions we have

$$\frac{\partial\psi^*}{\partial y^*} = cx^*, \quad \frac{\partial\psi^*}{\partial x^*} = 0, \quad \theta^* = 1 \quad \text{at } y^* = 0 \tag{18}$$

and

$$\frac{\partial\psi^*}{\partial y^*} \to 0, \quad \theta^* \to 0 \quad \text{as } y^* \to \infty. \tag{19}$$

Thus the set of transformations Γ reduces to a one parameter group of transformations as

$$x^* = xe^{\epsilon\alpha_1}, \quad y^* = y, \quad \psi^* = \psi e^{\epsilon\alpha_1},$$
$$u^* = ue^{\epsilon\alpha_1}, \quad v^* = v, \quad \theta^* = \theta. \tag{20}$$

2.4. Absolute invariants

First we find an absolute invariant which is a function of the dependent variable, namely η and $\eta = yx^s$.

For this purpose we write

$$x^* = Bx, \quad B = e^{\epsilon\alpha_1}, \quad y^* = B^{\frac{\alpha_2}{\alpha_1}}y, \quad \psi^* = B^{\frac{\alpha_3}{\alpha_1}}\psi. \tag{21}$$

To establish $y^*x^{*s} = yx^s$, we have $y^*x^{*s} = yB^{\frac{\alpha_2}{\alpha_1}}B^s x^s = yx^s B^{s+\frac{\alpha_2}{\alpha_1}}$.

Putting $s + \frac{\alpha_2}{\alpha_1} = 0$ we get, $y^*x^{*s} = yx^s$. Since $\alpha_2 = 0$ we have $s = 0$ and $\eta = y^*$. Thus

$$\eta = y^* \tag{22}$$

is an absolute invariant.

We now calculate a second absolute invariant G, which involves the dependent variable ψ. Let us assume that $G = x^{*r}\psi^*$.

Now, $x^{*r}\psi^* = B^r x^r B^{\frac{\alpha_3}{\alpha_1}}\psi = B^{r+\frac{\alpha_3}{\alpha_1}}x^r\psi$.

Putting $r + \frac{\alpha_3}{\alpha_1} = 0$, we have, $r = -\frac{\alpha_3}{\alpha_1} = -1$ (since $\alpha_1 = \alpha_3$). Thus, we get the second absolute invariant G as $G = x^{*-1}\psi^*$.

Putting $G = F(\eta)$ we can write

$$\psi^* = x^*F(\eta). \tag{23}$$

We also have $\theta^* = \theta(\eta)$.

In view of the relations for y^* and ψ^*, Eqs. (14) and (15) become

$$F'^2 - FF'' = -Av^*\theta' F'' + v^*(a + A - A\theta)F''' - cM^2F' \tag{24}$$

and

$$F\theta' + \kappa\theta'' = 0. \tag{25}$$

The boundary conditions are transformed as

$$F'(\eta) = c, \quad F(\eta) = 0 \quad \text{and } \theta(\eta) = 1 \quad \text{at } \eta = 0, \tag{26}$$
$$F'(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as } \eta \to \infty. \tag{27}$$

Again, to remove the constants c and v^* we introduce the following transformations for F, η and θ in Eqs. (24) and (25).

$$\eta = v^{*\alpha}c^{\beta}\eta^*, \quad F = v^{*\alpha'}c^{\beta'}F^*, \quad \theta = v^{*\alpha''}c^{\beta''}\bar{\theta}. \tag{28}$$

Using (28) in (24) and (25), we get

$$v^{*(2\alpha'-2\alpha)}c^{(2\beta'-2\beta)}F^{*\prime 2} - v^{*(2\alpha'-2\alpha)}c^{(2\beta'-2\beta)}F^*F^{*\prime\prime}$$
$$= -Av^{*(\alpha''-3\alpha+1)}c^{(\beta''+\beta'-2\beta)}\bar{\theta}'F^{*\prime\prime} + (a+A)v^{*(\alpha'-3\alpha+1)}$$
$$\times c^{(\beta'-3\beta)}F^{*\prime\prime\prime} - Av^{*(\alpha''+\alpha'-3\alpha+1)}c^{(\beta''+\beta'-3\beta)}F^{*\prime\prime\prime}\bar{\theta}$$
$$- M^2v^{*(\alpha'-\alpha)}c^{(\beta'-\beta+1)}F^{*\prime}, \tag{29}$$

$$(v^{*\alpha'}c^{\beta'}F^*)(v^{*(\alpha''-\alpha)}c^{(\beta''-\beta)}\bar{\theta}') + \kappa(v^{*(\alpha''-2\alpha)}c^{(\beta''-2\beta)}\bar{\theta}'') = 0. \tag{30}$$

Putting $2\alpha' - 2\alpha = \alpha'' + \alpha' - 3\alpha + 1 = \alpha' - 3\alpha + 1 = \alpha' - \alpha$, we get $\alpha = \alpha'$, $\alpha = \frac{1}{2}$, $\alpha'' = 0$.

Again putting $2\beta' - 2\beta = \beta'' + \beta' - 3\beta = \beta'' + \beta' - 3\beta = \beta' - 3\beta = \beta' - \beta + 1$, we get $\beta = -\frac{1}{2}$, $\beta' = -\beta = \frac{1}{2}$, $\beta'' = 0$.

Eqs. (27) and (28) become

$$F^{*\prime 2} - F^*F^{*\prime\prime} = -A\bar{\theta}'F^{*\prime\prime} + (a + A - A\bar{\theta})F^{*\prime\prime\prime} - M^2F^{*\prime}, \tag{31}$$

$$\bar{\theta}'' + PrF^*\bar{\theta}' = 0, \tag{32}$$

where $Pr = \frac{v^*}{\kappa}$.

The boundary conditions are

$$F^{*\prime} = 1, \quad F^* = 0, \quad \bar{\theta} = 1 \quad \text{at } \eta = 0, \tag{33}$$
$$F^{*\prime} \to 0, \quad \bar{\theta} \to 0 \quad \text{as } \eta \to \infty. \tag{34}$$

Taking $F^* = f$ and $\bar{\theta} = \theta$ the above Eqs. (31) and (32) take the following form:

$$f'^2 - ff'' = -A\theta' f'' + (a + A - A\theta)f''' - M^2f', \tag{35}$$
$$\theta'' + Prf\theta' = 0. \tag{36}$$

The boundary conditions are

$$f' = 1, \quad f = 0, \quad \theta = 1 \quad \text{at } \eta = 0 \quad \text{and}$$
$$f' \to 0, \quad \theta \to 0 \quad \text{as } \eta \to \infty. \tag{37}$$

3. Numerical method for solution

The above Eqs. (35) and (36) along with boundary conditions are solved by converting it to an initial value problem. We set

$$f' = z, \quad z' = p, \quad p' = \frac{(z^2 - fp + Aqp + M^2z)}{(a + A - A\theta)}, \tag{38}$$

$$\theta' = q, \quad q' = -Prfq \tag{39}$$

and the boundary conditions are

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1. \tag{40}$$

Since these equations are non-linear we can not superpose solutions on this problem. Furthermore, in order to integrate (38) and (39) as an initial value problem we require a value for $p(0)$ i.e. $f''(0)$ and $q(0)$ i.e. $\theta'(0)$ but no such values are given. The suitable guess values for $f''(0)$ and $\theta'(0)$ are chosen and then integration is carried out. We compare the calculated values for f' and θ at $\eta = 5$ (say) with the given boundary condition $f'(5) = 0$ and $\theta(5) = 0$ and adjust the estimated value, $f''(0)$ and $\theta'(0)$, to give a better approximation for the solution.

We take a series of values for $f''(0)$ and $\theta'(0)$ and apply the fourth-order classical Runge–Kutta method with step-size $h = 0.02$. To improve the solutions we use linear interpolation namely Secant method. The above procedure is repeated until we get the results upto the desired degree of accuracy, 10^{-5}.

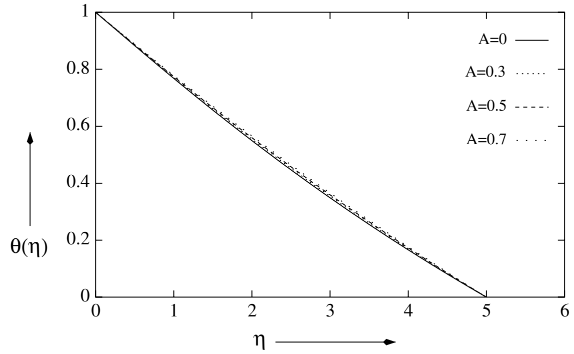

Fig. 2. Distribution of temperature $\theta(\eta)$ against η when $M = 0$, $Pr = 0.1$.

4. Results and discussions

In order to analyse the results, the numerical computation has been carried out using the method described in the previous section for various values of the parameter such as fluid viscosity variation parameter A, Hartman number M and Prandtl number Pr. For illustration of the results numerical values are plotted in the figures one to six. The physical explanation of the appropriate change of parameters are given below.

Fluid viscosity and thermal conductivity (hence thermal diffusivity) play an important role in the flow characteristics of laminar boundary layer problems. Fluid properties are significantly affected by the variation of temperature. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so the heat transfer rate at the wall is also affected.

First we concentrate in the velocity distribution and heat transfer with Prandtl number $Pr = 0.1$ in the absence of magnetic field ($M = 0$) and presented in Figs. 1 and 2. The horizontal velocity profiles in the sheet for the various values of $A(=0, 0.3, 0.5)$ are shown in Fig. 1. With the increasing A the thickness of the velocity boundary layer increases. So the velocity decreases with the increase of A, i.e. with the decreasing viscosity. Fig. 2 represents the temperature profiles for the same set of values of the parameter A. For any value of A considered, the temperature (θ) is found to decrease with the increase of η but the change of θ is not significant. The increase of temperature dependent fluid viscosity parameter (A) makes decrease of thermal boundary layer thickness, which results in decrease of temperature profile $\theta(\eta)$. Due to this reason, the horizontal velocity $f'(\eta)$ decreases in Fig. 1 and from these two figures it is clear that at a far distance from the sheet(here $\eta = 5$) the velocity as well as temperature vanishes.

Control of boundary layer flow is of practical significance. Several methods have been developed for the purpose of artificially controlling the behaviour of the boundary layer. The application of magnetohydrodynamic (MHD) principle is another method for affecting the flow field in the desired direction by altering the structure of the boundary layer.

Fig. 3 shows the horizontal velocity profile for various Hartmann number $M(=0, 1, 2)$ with constant Pr

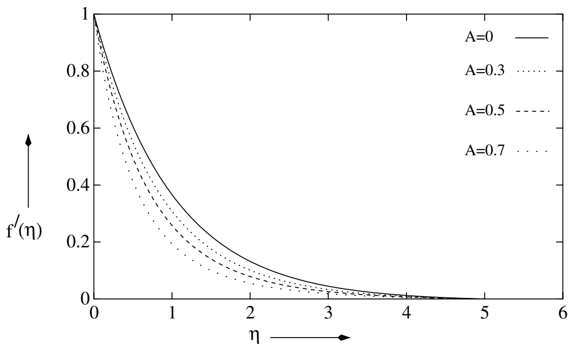

Fig. 1. Distribution of velocity $f'(\eta)$ against η when $M = 0$, $Pr = 0.1$.

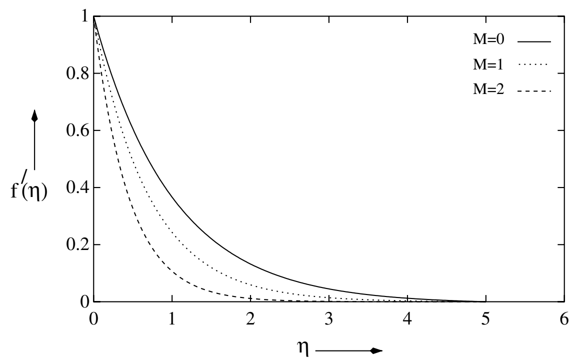

Fig. 3. Distribution of horizontal velocity $f'(\eta)$ against η when $A = 0$, $Pr = 0.1$.

(=0.1) the viscosity of the fluid being assumed uniform (i.e. $a = 1, A = 0$). The velocity curves show that the rate of transport is considerably reduced with the increase of M. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is due to the fact that variation of the Hartmann number leads to the variation of the Lorentz force due to magnetic field and the Lorentz force produces more resistance to transport phenomena. In all cases the velocity vanishes at some large distance from the sheet.

Fig. 4 exhibits the temperature profiles for different values of M $(= 0, 1, 2)$. In each case, temperature is found to decrease with the increase of η until it vanishes at $\eta = 5$. But the temperature is found to increase for any non-zero fixed value of η with the increase of M.

Fig. 5 shows the effects of Prandtl number (Pr) on the temperature $\theta(\eta)$ for fixed values of A and M. As anticipated, the thermal boundary layer thickness decreases with increasing Prandtl number (i.e. with the decreasing thermal diffusivity). It is clear from Fig. 5 that the temperature at a point decreases with increase in Prandtl number Pr but the increase in Prandtl number Pr has no such effect on the horizontal velocity. The increase of Prandtl number Pr means that the thermal diffusivity (as $A = 0$) decreases. So the rate of heat transfer is decreased due to the decrease of thermal boundary layer (Fig. 6).

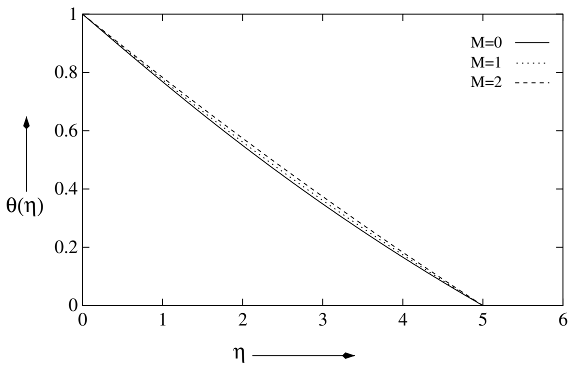

Fig. 4. Distribution of temperature $\theta(\eta)$ against η when $A = 0$, $Pr = 0.1$.

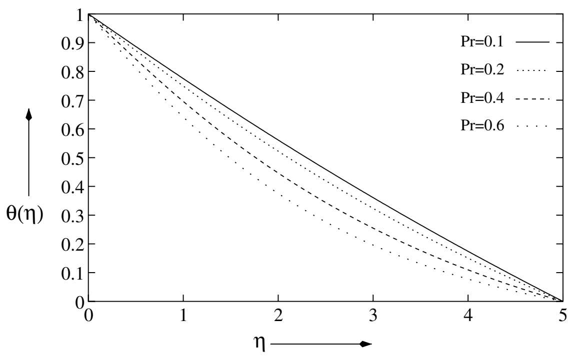

Fig. 5. Distribution of temperature $\theta(\eta)$ against η when $A = 0$, $M = 1$.

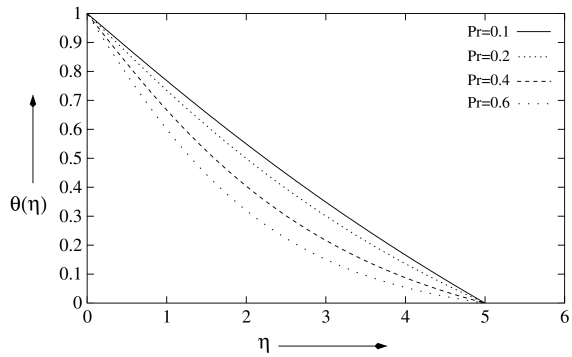

Fig. 6. Distribution of temperature $\theta(\eta)$ against η when $A = 0$, $M = 0$.

5. Conclusion

Under the assumption of temperature dependent viscosity, the present method gives solutions, for steady incompressible boundary layer flow over a heated stretching surface in the presence of uniform transverse magnetic field. The results pertaining to the present study indicate that the temperature dependent fluid viscosity plays a significant role in shifting the fluid away from the wall. The effect of transverse magnetic field on a viscous incompressible conducting fluid is to suppress the velocity field which in turn causes the enhancement of the temperature field.

Acknowledgments

One of the authors (S.M.) gratefully acknowledges the financial support of Council of Scientific and Industrial Research (CSIR), Delhi, India for pursuing this work.

The authors are thankful to the honourable reviewers for constructive suggestions.

References

[1] B.C. Sakiadis, Boundary-layer behaviour on continuous solid surface: I. The boundary-layer equations for two-dimensional and asymmetric flow, AIChE J. 7 (1961) 26.

[2] B.C. Sakiadis, Boundary-layer behaviour on continuous solid surface: II. The boundary-layer on a continuous flat surface, AIChE J. 7 (1961) 221.
[3] G.K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, London, 1967, p. 597.
[4] G.W. Bluman, S. Kumei, Symmetries and Differential Equations, Springer-Verlag, New York, 1989.
[5] L.J. Crane, Flow past a stretching plate, Z. Angew. Math. Phys. 21 (1970) 645–647.
[6] M. Pakdemirli, M. Yurusoy, Similarity transformations for partial differential equations, SIAM Rev. 40 (1998) 96–101.
[7] M. Oberlack, Similarity in non-rotating and rotating turbulent pipe flows, J. Fluid Mech. 379 (1999) 1–22.
[8] M.S. Abel, S.K. Khan, K.V. Prasad, Study of visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity, Int. J. Non-Linear Mech. 37 (2002) 81–88.
[9] P.S. Gupta, A.S. Gupta, Heat and mass transfer on a stretching sheet with suction and blowing, Can. J. Chem. Eng. 55 (1977) 744–746.
[10] P. Saikrishnan, S. Roy, Non-uniform slot injection (suction) into water boundary layers over (i) a cylinder and (ii) a sphere, Int. J. Eng. Sci. 41 (2003) 1351–1365.
[11] R.B. Bird, W.E. Stewart, E.N. Lightfoot, Transport Phenomena, John Wiley and Sons, New York, 1960.
[12] S. Roy, P. Saikrishnan, Non-uniform slot injection (suction) into steady laminar water boundary layer flow over a rotating sphere, Int. J. Heat Mass Transfer 46 (2003) 3389–3396.
[13] T. Tapanidis, Gr. Tsagas, H.P. Mazumdar, Application of scaling group of transformations to viscoelastic second-grade fluid flow, Nonlinear Funct. Anal. Appl. 8 (3) (2003) 345–350.